D^0–$\overline{D}^0$ MIXING

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The detailed formalism for $D^0 - \overline{D}^0$ mixing is presented in the "Note on *CP* Violation in Meson Decays" in this *Review*. For completeness, we present an overview here. The time evolution of the D^0–$\overline{D}^0$ system is described by the Schrödinger equation

$$i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} = \left(\mathbf{M} - \tfrac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix}, \tag{1}$$

where the $\mathbf{M}$ and $\mathbf{\Gamma}$ matrices are Hermitian, and CPT invariance requires that $M_{11} = M_{22} \equiv M$ and $\Gamma_{11} = \Gamma_{22} \equiv \Gamma$. The off-diagonal elements of these matrices describe the dispersive and absorptive parts of the mixing.

Because CP violation is expected to be quite small here, it is convenient to label the mass eigenstates by the CP quantum number in the limit of CP conservation. Thus, we write

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle\,, \tag{2}$$

where

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\,, \tag{3}$$

with the normalization condition that $|p|^2 + |q|^2 = 1$ and the sign is chosen so that D_1 has CP even, or nearly so. The corresponding eigenvalues are

$$\omega_{1,2} \equiv m_{1,2} - \tfrac{i}{2}\Gamma_{1,2} = \left(M - \tfrac{i}{2}\Gamma\right) \pm \frac{q}{p}\left(M_{12} - \tfrac{i}{2}\Gamma_{12}\right)\,, \tag{4}$$

where $m_{1,2}$ and $\Gamma_{1,2}$ are the masses and widths of the $D_{1,2}$.

We define reduced mixing parameters x and y by

$$x \equiv (m_1 - m_2)/\Gamma = \Delta m/\Gamma \tag{5}$$

and

$$y \equiv (\Gamma_1 - \Gamma_2)/2\Gamma = \Delta\Gamma/2\Gamma\,, \tag{6}$$

where $\Gamma \equiv (\Gamma_1 + \Gamma_2)/2$. We choose the phase convention $CP|D^0\rangle = +|\overline{D}^0\rangle$. If CP is conserved, then M_{12} and Γ_{12} are real and $\Delta m = 2M_{12}$, $\Delta\Gamma = 2\Gamma_{12}$. The signs of Δm and $\Delta\Gamma$ are to be determined experimentally.

The parameters x and y are measured in several ways. The most precise constraints are obtained using the time-dependence of D decays. Since D^0–$\overline{D}^0$ mixing is a small effect, the identifying tag of the initial particle as a D^0 or a $\overline{D}^0$ must be extremely accurate. The usual tag is the charge of the distinctive slow pion in the decay sequence $D^{*+} \rightarrow D^0\pi^+$ or $D^{*-} \rightarrow \overline{D}^0\pi^-$. In current experiments, the probability of mistagging is about 0.1%. The large data samples available from the B-factories allow the production flavor to also be determined by fully reconstructing charm on the "other side" of the event—significantly reducing the mistag rate. Another tag of comparable accuracy is identification of one of the D's produced from $\psi(3770) \rightarrow D^0\overline{D}^0$. Although time-dependent analyses are not possible at symmetric charm-threshold facilities (the D^0 and $\overline{D}^0$ do not travel far enough), the quantum-coherent $C = -1$ $\psi(3770) \rightarrow D^0\overline{D}^0$ state provides time-integrated sensitivity [1,2].

Time-Dependent Analyses: We extend the formalism of this *Review*'s note on "B^0–$\overline{B}^0$ Mixing" [3]. In addition to the "right-sign" instantaneous decay amplitudes $\overline{A}_f \equiv \langle f|H|\overline{D}^0\rangle$ and $A_{\overline{f}} \equiv \langle \overline{f}|H|D^0\rangle$ for CP conjugate final states $f = K^+\pi^-, ...$ and $\overline{f} = K^-\pi^+, ...$, we include "wrong-sign" amplitudes $\overline{A}_{\overline{f}} \equiv \langle \overline{f}|H|\overline{D}^0\rangle$ and $A_f \equiv \langle f|H|D^0\rangle$.

It is conventional to normalize the wrong-sign decay distributions to the integrated rate of right-sign decays and to express time in units of the precisely measured neutral D-meson mean lifetime, $\overline{\tau}_{D^0} = 1/\Gamma = 2/(\Gamma_1 + \Gamma_2)$. Starting from a pure $|D^0\rangle$ or $|\overline{D}^0\rangle$ state at $t = 0$, the time-dependent rates of decay to wrong-sign final states relative to the integrated right-sign decay rates are, to leading order:

$$r(t) \equiv \frac{\left|\langle f|H|D^0(t)\rangle\right|^2}{\left|\overline{A}_f\right|^2} = \left|\frac{q}{p}\right|^2 \left|g_+(t)\,\lambda_f^{-1} + \,g_-(t)\right|^2 , \qquad (7)$$

and

$$\overline{r}(t) \equiv \frac{\left|\langle \overline{f}|H|\overline{D}^0(t)\rangle\right|^2}{\left|A_{\overline{f}}\right|^2} = \left|\frac{p}{q}\right|^2 \left|g_+(t)\,\lambda_{\overline{f}} + \,g_-(t)\right|^2 . \qquad (8)$$

where

$$\lambda_f \equiv q\overline{A}_f/pA_f\,, \quad \lambda_{\bar{f}} \equiv q\overline{A}_{\bar{f}}/pA_{\bar{f}}\,, \qquad (9)$$

and

$$g_{\pm}(t) = \frac{1}{2}\left(e^{-iz_1 t} \pm e^{-iz_2 t}\right)\,, \quad z_{1,2} = \frac{\omega_{1,2}}{\Gamma}\,. \tag{10}$$

Note that a change in the convention for the relative phase of D^0 and $\overline{D}^0$ would cancel between q/p and $\overline{A}_f/A_f$ and leave λ_f unchanged. We expand $r(t)$ and $\overline{r}(t)$ to second order in x and y for modes in which the ratio of decay amplitudes, $R_D = |A_f/\overline{A}_f|^2$, is very small.

Semileptonic decays: In semileptonic D decays, $A_f = \overline{A}_{\overline{f}} = 0$ in the Standard Model, and $r(t)$ is

$$r(t) = |g_-(t)|^2 \left|\frac{q}{p}\right|^2 \approx \frac{e^{-t}}{4}(x^2+y^2)\,t^2 \left|\frac{q}{p}\right|^2 . \tag{11}$$

For $\overline{r}(t)$ one replaces q/p here with p/q. In the Standard Model, CP violation in charm mixing is small and $|q/p| \approx 1$. In the limit of CP conservation, $r(t) = \overline{r}(t)$, and the time-integrated mixing rate relative to the time-integrated right-sign decay rate is

$$R_M = \int_0^\infty r(t)dt = \frac{1}{2}(x^2+y^2)\,. \tag{12}$$

Table 1 summarizes results for R_M from semileptonic decays; the world average from the Heavy Flavor Averaging Group (HFAG) [10] is $R_M = (1.7 \pm 3.9) \times 10^{-4}$.

Table 1: Results for R_M in D^0 semileptonic decays.

Year	Exper.	Final state(s)	R_M $(\times 10^{-3})$	90% C.L.
2007	BABAR [4]	$K^{(*)+}e^-\overline{\nu}_e$	$0.04^{+0.70}_{-0.60}$	$< 1.2 \times 10^{-3}$
2005	Belle [5]	$K^{(*)+}e^-\overline{\nu}_e$	$0.02{\pm}0.47{\pm}0.14$	$< 1.0 \times 10^{-3}$
2005	CLEO [6]	$K^{(*)+}e^-\overline{\nu}_e$	$1.6{\pm}2.9{\pm}2.9$	$< 7.8 \times 10^{-3}$
2004*	BABAR [7]	$K^{(*)+}e^-\overline{\nu}_e$	$2.3{\pm}1.2{\pm}0.4$	$< 4.2 \times 10^{-3}$
2002*	FOCUS [8]	$K^+\mu^-\overline{\nu}_\mu$	$-0.76^{+0.99}_{-0.93}$	$< 1.01 \times 10^{-3}$
1996	E791 [9]	$K^+\ell^-\overline{\nu}_\ell$	$(1.1^{+3.0}_{-2.7}) \times 10^{-3}$	$< 5.0 \times 10^{-3}$
	HFAG [10]		0.17 ± 0.39	

*These measurements are excluded from the HFAG average. The FOCUS result is unpublished, and the BABAR result has been superseded by Ref. 4.

Wrong-sign decays to hadronic non-CP eigenstates: Consider the final state $f = K^+\pi^-$, where A_f is doubly Cabibbo-suppressed. The ratio of decay amplitudes is

$$\frac{A_f}{\overline{A}_f} = -\sqrt{R_D}\, e^{-i\delta_f}, \qquad \left|\frac{A_f}{\overline{A}_f}\right| \sim O(\tan^2\theta_c)\,, \tag{13}$$

where R_D is the doubly Cabibbo-suppressed (DCS) decay rate relative to the Cabibbo-favored (CF) rate, δ_f is the strong phase difference between DCS and CF processes, and θ_c is the Cabibbo angle. The minus sign originates from the sign of V_{us} relative to V_{cd}.

We characterize the violation of CP in the mixing amplitude, the decay amplitude, and the interference between mixing and decay, by real-valued parameters A_M, A_D, and ϕ. We adopt the parametrization (see Refs. 12 and 13).

$$\left|\frac{q}{p}\right|^2 = \sqrt{\frac{1+A_M}{1-A_M}}\,, \tag{14}$$

$$\lambda_f^{-1} \equiv \frac{pA_f}{q\overline{A}_f} = -\sqrt{R_D}\left(\frac{(1+A_D)(1-A_M)}{(1-A_D)(1+A_M)}\right)^{1/4} e^{-i(\delta_f+\phi)}\,, \tag{15}$$

$$\lambda_{\overline{f}} \equiv \frac{q\overline{A}_{\overline{f}}}{pA_{\overline{f}}} = -\sqrt{R_D}\left(\frac{(1-A_D)(1+A_M)}{(1+A_D)(1-A_M)}\right)^{1/4} e^{-i(\delta_f-\phi)}\,. \tag{16}$$

Since

$$\sqrt{\frac{1+A_D}{1-A_D}} = \frac{|A_f/\overline{A}_f|}{|\overline{A}_{\overline{f}}/A_{\overline{f}}|}\,, \tag{17}$$

A_D is a measure of direct CP violation, while A_M is a measure of CP violation in mixing. The angle ϕ measures CP violation in interference between mixing and decay. While A_M is independent of the decay process, A_D and ϕ may depend on f.

In general, $\lambda_{\overline{f}}$ and λ_f^{-1} are independent complex numbers. More detail on CP violation in meson decays can be found in Ref. 3. To leading order, for A_D and $A_M \ll 1$,

$$r(t) = e^{-t}\Big[R_D(1+A_D) + \sqrt{R_D}\sqrt{1+A_M}\sqrt{1+A_D}\,y'_-t + \frac{1}{2}(1+A_M)R_M\, t^2\Big] \tag{18}$$

and

$$\overline{r}(t) = e^{-t}\Big[R_D(1-A_D) + \sqrt{R_D}\sqrt{1-A_M}\sqrt{1-A_D}y'_{-}t$$
$$+\frac{1}{2}(1-A_M)R_M\,t^2\Big] \tag{19}$$

Here

$$y'_{\pm} \equiv\; y'\cos\phi \pm x'\sin\phi$$
$$= y\cos(\delta_{K\pi}\mp\phi) - x\sin(\delta_{K\pi}\mp\phi)\ , \tag{20}$$

where

$$y' \equiv\; y\cos\delta_{K\pi} - x\sin\delta_{K\pi},$$
$$x' \equiv x\cos\delta_{K\pi} + y\sin\delta_{K\pi}\ , \tag{21}$$

and $R_M \approx \left(x^2+y^2\right)/2 = \left(x'^2+y'^2\right)/2$ is the mixing rate relative to the time-integrated right-sign rate.

The three terms in Eq. (18) and Eq. (19) probe the three fundamental types of CP violation. In the limit of CP conservation, A_M, A_D, and ϕ are all zero. Then

$$r(t) = \overline{r}(t) = e^{-t}\left(R_D + \sqrt{R_D}\,y't + \frac{1}{2}R_M\,t^2\right)\ , \tag{22}$$

and the time-integrated wrong-sign rate relative to the integrated right-sign rate is

$$R = \int_0^\infty r(t)\,dt = R_D + \sqrt{R_D}\,y' + R_M\ . \tag{23}$$

The ratio R is the most readily accessible experimental quantity. Table 2 gives recent measurements of R in $D^0 \to K^+\pi^-$ decay. The average R is $(0.380 \pm 0.005)\,\%$.

Table 2 gives the limits on A_D, and the HFAG average [10] of R_D and A_D from a general fit; all allow for both mixing and CP violation. Typically, the fit parameters are R_D, x'^2, and y'. Table 3 summarizes the results for y' and x'^2. Allowing for CP violation, the separate contributions to R can be extracted by fitting the $D^0 \to K^+\pi^-$ and $\overline{D}^0 \to K^-\pi^+$ decay rates.

Table 4 summarizes results for R measured in multibody final states with nonzero strangeness. Here R, defined in Eq. (23), becomes an average over the Dalitz plot.

Table 2: Results for R, R_D, and A_D in $D^0 \to K^+\pi^-$.

Year	Exper.	$R(\times 10^{-3})$	$R_D(\times 10^{-3})$	$A_D(\%)$
2007	CDF [14]	4.15 ± 0.10	3.04 ± 0.55	—
2007	BABAR [15]	$3.53 \pm 0.08 \pm 0.04$	$3.03 \pm 0.16 \pm 0.10$	$-2.1 \pm 5.2 \pm 1.5$
2006	Belle [16]	$3.77 \pm 0.08 \pm 0.05$	3.64 ± 0.17	2.3 ± 4.7
2005*	FOCUS [17]	$4.29^{+0.63}_{-0.61} \pm 0.28$	$5.17^{+1.47}_{-1.58} \pm 0.76$	$13^{+33}_{-25} \pm 10$
2000*	CLEO [11]	$3.32^{+0.63}_{-0.65} \pm 0.40$	$4.8 \pm 1.2 \pm 0.4$	$-1^{+16}_{-17} \pm 1$
1998	E791 [18]	$6.8^{+3.4}_{-3.3} \pm 0.7$	—	—
	Average	3.80 ± 0.05	3.35 ± 0.09 [10]	-2.2 ± 2.5 [10]

*These measurements are included in the HFAG average of R_D but are excluded from the HFAG average A_D.

Table 3: Results on the time-dependence of $r(t)$ in $D^0 \to K^+\pi^-$ and $\overline{D}^0 \to K^-\pi^+$ decays. The CDF result assumes no CP violation. The FOCUS and CLEO results restrict x'^2 to the physical region. The confidence intervals from FOCUS and CLEO are obtained from the fit, whereas Belle uses a Feldman-Cousins method, and CDF uses a Bayesian method.

Year	Exper.	y' (%)	$x'^{\,2}$ $(\times 10^{-3})$
2007	CDF [14]	0.85 ± 0.76	-0.12 ± 0.35
2007	BABAR [15]	$0.97 \pm 0.44 \pm 0.31$	$-0.22 \pm 0.30 \pm 0.21$
2006	Belle [16]	$-2.8 < y' < 2.1$	< 0.72 (95% C.L.)
2005	FOCUS [17]	$-11.2 < y' < 6.7$	< 8.0 (95% C.L.)
2000	CLEO [11]	$-5.8 < y' < 1.0$	< 0.81 (95% C.L.)

Extraction of the mixing parameters x and y from the results in Table 3 requires knowledge of the relative strong phase $\delta_{K\pi}$. An interference effect that provides useful sensitivity to $\delta_{K\pi}$ arises in the decay chain $\psi(3770) \to D^0\overline{D}^0 \to (f_{cp})(K^+\pi^-)$, where f_{cp} denotes a CP-even or -odd eigenstate from D^0 decay, such as K^+K^- [26]. Here, the amplitude relation

$$\sqrt{2}\, A(D_\pm \to K^-\pi^+) = A(D^0 \to K^-\pi^+) \pm A(\overline{D}^0 \to K^-\pi^+). \tag{24}$$

Table 4: Results for R in $D^0 \to K^{(*)+}\pi^-(n\pi)$. The values of R need not be the same for different decay channels.

Year	Exper.	D^0 final state	$R(\%)$
2006	BABAR [22]	$K^+\pi^-\pi^0$	$0.214 \pm 0.008 \pm 0.008$
2005	Belle [23]	$K^+\pi^-\pi^+\pi^-$	$0.320 \pm 0.018^{+0.018}_{-0.013}$
2005	Belle [23]	$K^+\pi^-\pi^0$	$0.229 \pm 0.015^{+0.013}_{-0.009}$
2002	CLEO [19]	$K^{*+}\pi^-$	$0.5 \pm 0.2^{+0.6}_{-0.1}$
2001	CLEO [24]	$K^+\pi^-\pi^+\pi^-$	$0.41^{+0.12}_{-0.11} \pm 0.04$
2001	CLEO [25]	$K^+\pi^-\pi^0$	$0.43^{+0.11}_{-0.10} \pm 0.07$
1998	E791 [18]	$K^+\pi^-\pi^+\pi^-$	$0.68^{+0.34}_{-0.33} \pm 0.07$

where $D_\pm$ denotes a CP-even or -odd eigenstate, implies that

$$\cos\delta_{K\pi} = \frac{|A(D_+ \to K^-\pi^+)|^2 - |A(D_- \to K^-\pi^+)|^2}{2\sqrt{R_D}\,|A(D^0 \to K^-\pi^+)|^2}\,. \quad (25)$$

This neglects CP violation and uses $\sqrt{R_D} \ll 1$.

For multibody final states, Eqs. (13)–(23) apply separately to each point in phase-space. Although x and y do not vary across the space, knowledge of the resonant substructure is needed to extrapolate the strong phase difference δ from point to point to determine x and y.

A time-dependent Dalitz-plot analysis of $D^0 \to K^+\pi^-\pi^0$ from BABAR [22,28] reports $R_M = (2.9 \pm 1.6) \times 10^{-4}$, $x'' = (2.39 \pm 0.61 \pm 0.32)\%$, and $y'' = (-0.14 \pm 0.60 \pm 0.40)\%$, where x'', y'', and $\delta_{K\pi\pi^0}$ are defined as

$$x'' \equiv x\cos\delta_{K\pi\pi^0} + y\sin\delta_{K\pi\pi^0}\,, y'' \equiv\ y\cos\delta_{K\pi\pi^0} - x\sin\delta_{K\pi\pi^0}, \quad (26)$$

in parallel to x', y', and $\delta_{K\pi}$ of Eq. (21). Both strong phases, $\delta_{K\pi}$ and $\delta_{K\pi\pi^0}$, can be determined from time-integrated CP asymmetries in correlated $D^0\overline{D}^0$ produced at the $\psi(3770)$ [26,27].

Both the sign and magnitude of x and y without phase or sign ambiguity may be measured using the time-dependent resonant substructure of multibody D^0 decays—see CLEO [29]. In $D^0 \to K^0_S\pi^+\pi^-$, the DCS and CF decay amplitudes populate the same Dalitz plot, which allows direct measurement of the relative strong phases. CLEO [19] and Belle [20] have measured the relative phase between $D^0 \to K^*(892)^+\pi^-$ and $D^0 \to K^*(892)^-\pi^+$ to be $(189 \pm 10 \pm 3^{+15}_{-5})^\circ$ and $(171.9 \pm 1.3$ (stat. only)$)^\circ$, respectively. These results are close to the 180° expected from Cabibbo factors and a small strong phase. Table 5 summarizes the results from Belle [20] of a time-dependent Dalitz-plot analysis of $D^0 \to K^0_S\pi^+\pi^-$.

Table 5: Belle results from a time-dependent Dalitz-plot analysis of $D^0 \to K^0_S\pi^+\pi^-$ [20]. The errors are statistical, experimental systematic, and decay-model systematic, respectively (CPV = CP violation).

Result	95% C.L. interval
No CP Violation	
$x = (0.80 \pm 0.29^{+0.09+0.10}_{-0.07-0.14})\%$	$(0.0, 1.6)\%$
$y = (0.33 \pm 0.24^{+0.08+0.06}_{-0.12-0.08})\%$	$(-0.34, 0.96)\%$
With CP Violation	
$x = (0.81 \pm 0.30^{+0.10+0.09}_{-0.07-0.16})\%$	$\|x\| < 1.6\%$
$y = (0.37 \pm 0.25^{+0.07+0.07}_{-0.13-0.08})\%$	$\|y\| < 1.04\%$
$\|q/p\| = 0.86^{+0.30+0.06}_{-0.29-0.03} \pm 0.08$	
$\phi = (-14^{+16+5+2}_{-18-3-4})^\circ$	

In addition, Belle [20] has reported results for both the relative phase (statisical errors only) and ratio R (central values only) of the DCS fit fraction relative to the CF fit fractions for $K^*(892)^+\pi^-$, $K_0^*(1430)^+\pi^-$, $K_2^*(1430)^+\pi^-$, $K^*(1410)^+\pi^-$, and $K^*(1680)^+\pi^-$. The reported values for R in units of $\tan^4\theta_c$ are 2.94 ± 0.12, 22.0 ± 1.6, 34 ± 4, 87 ± 13, and $(5 \pm 5) \times 10^2$. For $K^+\pi^-$, the corresponding value for R_D is $(1.28 \pm 0.02) \times \tan^4\theta_c$. Similarly, BABAR [21] has reported central values for R for $K^*(892)^+\pi^-$, $K_0^*(1430)^+\pi^-$, and $K_2^*(1430)^+\pi^-$. The reported values for R in units of $\tan^4\theta_c$ are 3.45 ± 0.31, 7.7 ± 3.0, and 1.7 ± 1.7, respectively. The systematic uncertainties on these value R must be evaluated. The large differences in R among these final states could point to an interesting role for hadronic effects.

Decays to CP Eigenstates: When the final state f is a CP eigenstate, there is no distinction between f and $\overline{f}$, and $A_f = A_{\overline{f}}$ and $\overline{A}_{\overline{f}} = \overline{A}_f$. We denote final states with CP eigenvalues ± 1 by $f_\pm$ and write $\lambda_\pm$ for $\lambda_{f_\pm}$.

The quantity y may be measured by comparing the rate for D^0 decays to non-CP eigenstates such as $K^-\pi^+$ with decays to CP eigenstates such as K^+K^- [13]. If decays to K^+K^- have a shorter effective lifetime than those to $K^-\pi^+$, y is positive.

In the limit of slow mixing, $x, y \ll 1$, and the absence of direct CP violation ($A_D = 0$), but allowing for small indirect CP violation ($|A_M|, |\phi| \ll 1$), we can write

$$\lambda_\pm = \left|\frac{q}{p}\right| e^{i\phi} \ . \tag{27}$$

To a good approximation, the decay rates for states that are initially D^0 and $\overline{D}^0$ to a CP eigenstate have exponential time dependence

$$r_\pm(t) \propto \exp\left(-t/\tau_\pm\right) \ , \tag{28}$$

$$\overline{r}_\pm(t) \propto \exp\left(-t/\overline{\tau}_\pm\right) \ , \tag{29}$$

where τ is measured in units of $1/\Gamma$.

The effective lifetimes are given by

$$1/\tau_\pm = 1 \pm \left|\frac{q}{p}\right| (y\cos\phi - x\sin\phi) \ , \tag{30}$$

$$1/\overline{\tau}_\pm = 1 \pm \left|\frac{p}{q}\right| (y\cos\phi + x\sin\phi) \ . \tag{31}$$

The effective decay rate to a CP eigenstate combining both D^0 and $\overline{D}^0$ decays is

$$r_\pm(t) + \overline{r}_\pm(t) \propto e^{-(1 \pm y_{CP})t} \ . \tag{32}$$

Here

$$y_{CP} = \frac{1}{2}\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) y\cos\phi - \frac{1}{2}\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) x\sin\phi \tag{33}$$

$$\approx y\cos\phi - A_M x\sin\phi \ . \tag{34}$$

If CP is conserved, $y_{CP} = y$.

Belle [30] and BaBar [31] have recently updated y_{CP} and the decay-rate asymmetry for CP even final states

$$A_\Gamma = \frac{\overline{\tau}_+ - \tau_+}{\overline{\tau}_+ + \tau_+} \tag{35}$$

$$= \frac{1}{2}\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) y\cos\phi - \frac{1}{2}\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) x\sin\phi \tag{36}$$

$$\approx A_M y\cos\phi - x\sin\phi \ . \tag{37}$$

If CP is conserved, $A_\Gamma = 0$. All measurements of y_{CP} and A_Γ are relative to the $D^0 \to K^-\pi^+$ decay rate. Table 6 summarizes the current status of measurements. The average of the six y_{CP} measurements is $1.13 \pm 0.27\%$.

Table 6: Results for y from $D^0 \to K^+K^-$ and $\pi^+\pi^-$.

Year	Exper.	D^0 final state(s)	$y(\%)$	$A_\Gamma(\times 10^{-3})$
2007	BABAR [31]	$K^+K^-, \pi^+\pi^-$	$1.03 \pm 0.33 \pm 0.19$	$2.6 \pm 3.6 \pm 0.8$
2007	Belle [30]	$K^+K^-, \pi^+\pi^-$	$1.31 \pm 0.32 \pm 0.25$	$0.1 \pm 3.0 \pm 1.5$
2001	CLEO [32]	$K^+K^-, \pi^+\pi^-$	$-1.2 \pm 2.5 \pm 1.4$	—
2001	Belle [33]	K^+K^-	$-0.5 \pm 1.0^{+0.7}_{-0.8}$	—
2000	FOCUS [34]	K^+K^-	$3.42 \pm 1.39 \pm 0.74$	—
1999	E791 [35]	K^+K^-	$0.8 \pm 2.9 \pm 1.0$	—
	HFAG Avg. [10]		1.132 ± 0.266	0.123 ± 0.248

Substantial work on the integrated CP asymmetries in decays to CP eigenstates indicates that A_{CP} is consistent with zero at the few-percent level [36]. The expression for the integrated CP asymmetry that includes the possibility of CP violation in mixing is

$$A_{CP} = \frac{\Gamma(D^0 \to f_\pm) - \Gamma(\overline{D}^0 \to f_\pm)}{\Gamma(D^0 \to f_\pm) + \Gamma(\overline{D}^0 \to f_\pm)} \tag{38}$$

$$= |q|^2 - |p|^2 + 2\mathrm{Re}\left(\frac{1 \mp \lambda_\pm}{1 \pm \lambda_\pm}\right) \ . \tag{39}$$

Coherent $D^0\overline{D}^0$ Analyses: Measurements of R_D, $\cos\delta_{K\pi}$, x, and y can be made simultaneously in a combined fit to the single-tag (ST) and double-tag (DT) yields, or individually by a series of "targeted" analyses [26,27].

The "comprehensive" analysis simultaneously measures mixing and DCS parameters by examining various ST and DT rates. Due to quantum correlations in the $C = -1$ and $C = +1$ $D^0\overline{D}^0$ pairs produced in the reactions $e^+e^- \to D^0\overline{D}^0(\pi^0)$ and $e^+e^- \to D^0\overline{D}^0\gamma(\pi^0)$, respectively, the time-integrated $D^0\overline{D}^0$ decay rates are sensitive to interference between amplitudes for indistinguishable final states. The size of this interference is governed by the relevant amplitude ratios and can include contributions from D^0–$\overline{D}^0$ mixing.

The following categories of final states are considered:

***f or $\bar{f}$*:** Hadronic states accessed from either D^0 or $\overline{D}^0$ decay but that are not CP eigenstates. An example is $K^-\pi^+$, which results from Cabibbo-favored D^0 transitions or DCS $\overline{D}^0$ transitions.

***ℓ^+ or ℓ^-*:** Semileptonic or purely leptonic final states, which, in the absence of mixing, tag unambiguously the flavor of the parent D^0.

***S_+ or S_-*:** CP-even and CP-odd eigenstates, respectively.

The decay rates for $D^0\overline{D}^0$ pairs to all possible combinations of the above categories of final states are calculated in Ref. 1, for both $C = -1$ and $C = +1$, reproducing the work of Ref. 2. Such $D^0\overline{D}^0$ combinations, where both D final states are specified, are double tags. In addition, the rates for single tags, where either the D^0 or $\overline{D}^0$ is identified and the other neutral D decays generically are given in Ref. 1.

CLEO-c has reported results using 281 pb^{-1} of $e^+e^- \to \psi(3770)$ data [37,38], where the quantum coherent $D^0\overline{D}^0$ pairs are in the $C = -1$ state. The values of y, R_M, and $\cos\delta_{K\pi}$ are determined from a combined fit to the ST (hadronic only) and DT yields. The hadronic final states included are $K^-\pi^+$ (f), $K^+\pi^-$ $(\bar{f})$, K^-K^+ $(S+)$, $\pi^+\pi^-$ (S_+), $K^0_S\pi^0\pi^0$ (S_+), $K^0_L\pi^0$ (S_+), $K^0_S\pi^0$ (S_-), $K^0_S\eta$ (S_-), and $K^0_S\omega$ (S_-). The two flavored final states, $K^-\pi^+$ and $K^+\pi^-$, can be reached via CF or DCS transitions.

Semileptonic DT yields are also included, where one D is fully reconstructed in one of the hadronic modes listed above, and the other D is partially reconstructed, requiring that only the electron be found. When the electron is accompanied by a flavor tag ($D \to K^-\pi^+$ or $K^+\pi^-$), only the "right-sign" DT sample, where the electron and kaon charges are the same, is used.

The main results of the CLEO-c analysis are the determination of $\cos\delta_{K\pi} = 1.10 \pm 0.35 \pm 0.07$, and World Averages for the mixing parameters from an "extended" fit that combines the CLEO-c data with previous mixing and branching-ratio measurements [37,38]. In these fits, which allow $\cos\delta_{K\pi}$ and x^2 to be unphysical, the no-mixing result ($x = y = 0$) is excluded at 5.0σ. Constraining $\cos\delta_{K\pi}$ and $\sin\delta_{K\pi}$ to [-1,+1]—that is interpreting $\delta_{K\pi}$ as an angle—yields $\delta_{K\pi} = (22^{+11+9}_{-12-11})^\circ$. Note that measurements of y (Table 6 and Table 3) and y' (Table 5) contribute to the determination of $\delta_{K\pi}$.

Summary of Experimental Results: Several recent results indicate that charm mixing is at the upper end of the range of Standard Model estimates.

BABAR [15] and CDF [14] find evidence for oscillations in $D^0 \to K^+\pi^-$ with 3.9σ ($\Delta\mathrm{Log}\mathcal{L}$) and 3.8σ (Bayesian), respectively. The most precise measurement is from Belle [16], which excludes $x'^2 = y' = 0$ at 2.1σ.

Belle [30] and BABAR [31] find 3.2σ and 3σ effects for y_{CP} in $D^0 \to K^+K^-$ and $\pi^+\pi^-$. The most sensitive measurement of y is in $D^0 \to K^0_S\pi^+\pi^-$ from Belle [20] and is only 1.2σ significant. In the same analysis, Belle also finds a 2.4σ result for x. The current situation would benefit from better knowledge of the strong phase difference $\delta_{K\pi}$ than provided by the current CLEO-c result [37,38]. This would allow one to unfold x and y from the $D^0 \to K^+\pi^-$ measurements of x'^2 and y', and directly compare them to the $D^0 \to K^0_S\pi^+\pi^-$ results.

The experimental data consistently indicate that the D^0 and $\overline{D}^0$ do mix. The mixing is presumably dominated by long-range processes. A serious limitation to the interpretation of charm oscillations in terms of New Physics is the theoretical uncertainty of the Standard Model prediction. However, recent evidence opens the window to searches for CP violation, which would provide unequivocal evidence of New Physics.

HFAG Averaging of Charm Mixing Results:

All mixing measurements can be combined to obtain world average values for x and y. The HFAG has done this in two ways [39,40,10]: (a) By adding together log-likelihood functions for x, y, and $\delta_{K\pi}$ from measurements of relevant observables; (b) by making a global fit to the measured observables x, y, $\delta_{K\pi}$, $\delta_{K\pi\pi^0}$, and R_D, being careful to account for the correlations among observables by using the error matrices from the experiments. Both methods use measurements of $D^0 \to K^+\ell^-\overline{\nu}$, K^+K^-, $\pi^+\pi^-$, $K^+\pi^-$, $K^+\pi^-\pi^0$, $K^+\pi^-\pi^+\pi^-$, and $K^0_S\pi^+\pi^-$ decays, as well as CLEO-c results for double-tagged branching fractions measured at the $\psi(3770)$ resonance (see previous sections of this note). Method (a) has the advantage that non-Gaussian errors are accounted for; method (b) has the advantage that it is readily expanded to allow for CP violation. For that, three additional parameters are included in the fit: $A_D \equiv (R^+_D - R^-_D)/(R^+_D + R^-_D)$, $|q/p|$, and $\mathrm{Arg}(q/p) \equiv \phi$. The two methods obtain almost identical results when they are applied to the same set of observables.

Table 7: HFAG Charm Mixing Average allowing for CP violation [10,39,40].

Parameter	HFAG average	95% C.L. interval
$x(\%)$	$0.97^{+0.27}_{-0.29}$	$(0.39 - 1.48)$
$y(\%)$	$0.78^{+0.18}_{-0.19}$	$(0.41 - 1.13)$
$R_D(\%)$	0.335 ± 0.009	$(0.316 - 0.353)$
$\delta_{K\pi}(^\circ)$	$21.9^{+11.5}_{-12.5}$	$(-6.3 - 44.6)$
$\delta_{K\pi\pi^0}(^\circ)$	$32.4^{+25.1}_{-25.8}$	$(-20.3 - 82.7)$
$A_D(\%)$	-2.2 ± 2.5	$(-7.10 - 2.67)$
$\|q/p\|$	$0.86^{+0.18}_{-0.15}$	$(0.59 - 1.23)$
$\phi(^\circ)$	$-9.6^{+8.3}_{-9.5}$	$(-30.3 - 6.5)$

For the global fit, confidence contours in the two dimensions (x, y) and $(|q/p|, \phi)$ are obtained by letting, for any point in the two-dimensional plane, all other fit parameters take their preferred values. Figures 1 and 2 show the resulting 1-to-5 σ contours. The fits exclude the no-mixing point $(x = y = 0)$ at

6.7σ, whether or not CP violation is allowed. The parameters x and y differ from zero by 3.0σ and 4.1σ, respectively. One-dimensional likelihood functions for parameters are obtained by allowing, for any value of the parameter, all other fit parameters to take their preferred values. The resulting likelihood functions give central values, 68.3% C.L. intervals, and 95% C.L. intervals as listed in Table 7.

From the results of the HFAG averaging, the following can be concluded: (1) Since CP violation is small and y_{CP} is positive, the CP-even state is shorter-lived, as in the $K^0\overline{K}^0$ system. However, since x appears to be positive, the CP-even state is heavier, unlike in the $K^0\overline{K}^0$ system. (2) There is no evidence yet for CP-violation in the $D^0\overline{D}^0$ system.

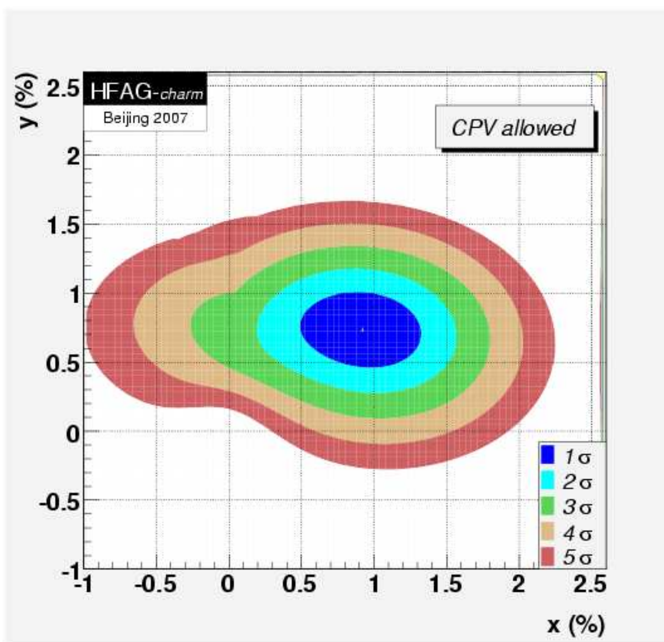

Figure 1: Two-dimensional 1σ-5σ contours for (x, y) from measurements of $D^0 \to K^+\ell\nu$, h^+h^-, $K^+\pi^-$, $K^+\pi^-\pi^0$, $K^+\pi^-\pi^+\pi^-$, and $K^0_S\pi^+\pi^-$ decays, and double-tagged branching fractions measured at the $\psi(3770)$ resonance (from HFAG [10]) .

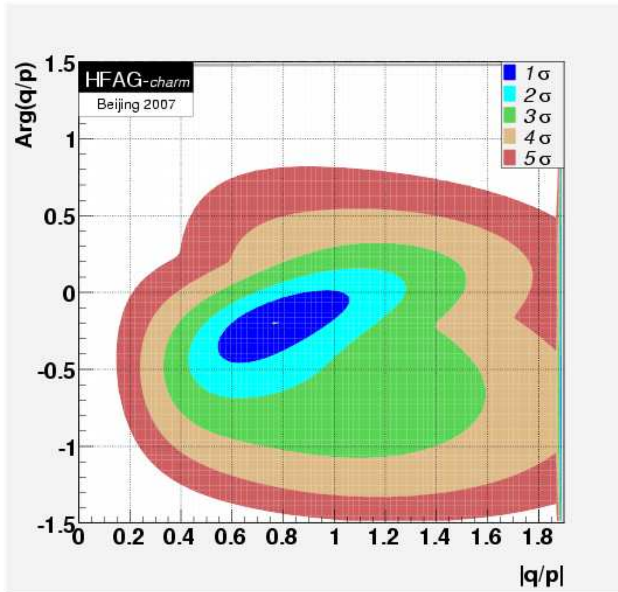

Figure 2: Two-dimensional 1σ-5σ contours for $(|q/p|,\mathrm{Arg}(q/p))$ from measurements of $D^0 \to K^+\ell\nu$, h^+h^-, $K^+\pi^-$, $K^+\pi^-\pi^0$, $K^+\pi^-\pi^+\pi^-$, and $K^0_S\pi^+\pi^-$ decays, and double-tagged branching fractions measured at the $\psi(3770)$ resonance (from HFAG [10]) .

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